DAY TWELVE

Transfer of Heat

Learning & Revision for the Day

- Heat
- Temperature
- Thermal Expansion
- Specific Heat
- Calorimetry
- Latent Heat
- Heat Transfer
- Perfectly Black Body
- Stefan's Law
- Newton's Law of Cooling
- Wein's Displacement Law
- Green House Effect

Heat

Heat is a form of energy which characterises the thermal state of matter. It is transferred from one body to the other due to temperature difference between them.

Heat is a scalar quantity with dimensions $[ML^2T^{-2}]$ and its SI unit is joule (J) while practical unit is calorie (cal); 1 cal = 4.18 J

If mechanical energy (work) is converted into heat then, the ratio of work done (W) to heat produced (Q) always remains the same and constant.

i.e. $$\frac{W}{Q} = \text{constant} = J \quad \text{or} \quad W = JQ$$

The constant J is called **mechanical equivalent of heat**.

Temperature

The factor that determines the flow of heat from one body to another when they are in contact with each other, is called temperature. Its SI unit is kelvin.

Thermometer

An instrument used to measure the temperature of a body is called a thermometer. For construction of thermometer, two fixed reference point **ice point** and **steam point** are taken. Some common types of thermometers are as follows:

1. **Liquid (mercury) thermometer** Range of temperature: –50°C to 350°C
2. **Gas thermometer (Nitrogen gas)** Range of temperature: –200°C to 1600°C
3. **Pyrometers** Range of temperature: –800°C to 6000°C

Scales of Temperature

Three most common scales are Celsius scale or Centigrade scale, Fahrenheit scale and Kelvin scale (Absolute scale).

Scale	Ice point / lower reference point	Steam point / Upper reference point	Unit
Celsius	0	100	°C
Fahrenheit	32	212	°F
Kelvin	273.15	373.15	K

Relation between C, F and K scales is

$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273.15}{5}$$

In general, $$\frac{\text{Temperature of } X - \text{Ice point of } X}{\text{Steam point of } X - \text{Ice point of } X} = \frac{\text{Temperature of } Y - \text{Ice point of } Y}{\text{Steam point of } Y - \text{Ice point of } Y}$$

Thermal Expansion

Almost all substances (solid, liquid and gas) expand on heating and contract on cooling. The expansion of a substance on heating is called thermal expansion of substance.

Thermal Expansion of Solids

Thermal expansion in solids is of three types:

1. **Linear Expansion** Thermal expansion along a single dimension of a solid body is defined as the linear expansion. If a rod is having length l_0 at temperature T, then elongation in length of rod due to rise in temperature by ΔT is,

   $$\Delta l = l_0 \alpha \Delta T \quad \text{or} \quad \alpha = \frac{\Delta l}{l_0 \times \Delta T}$$

   where, α is the coefficient of linear expansion whose value depends on the nature of the material.

   Final length, $l_f = l_0 + l_0 \alpha \Delta T = l_0(1 + \alpha \Delta T)$

2. **Superficial Expansion or Areal Expansion**

   Expansion of solids along two dimensions of solid objects is defined as superficial expansion.

   Coefficient of superficial expansion, $\beta = \dfrac{\Delta A}{A_0 \times \Delta T}$

   Final area, $A_f = A_0(1 + \beta \Delta T)$

   where, A_0 is the area of the body at temperature T.

3. **Volume or Cubical Expansion** Expansion of solids along three dimensions of solids objects is defined as cubical expansion.

   Coefficient of volume or cubical expansion, $\gamma = \dfrac{\Delta V}{V_0 \times \Delta T}$

   Final volume, $V = V_0(1 + \gamma \Delta T)$.

   where, V_0 is the volume of the body at temperature T.

NOTE
- The coefficients α, β and γ for a solid are related to each other.

  $$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

- As temperature increases, density decreases according to relation,

  $$\rho = \frac{\rho_0}{1 + \gamma \Delta T}$$

  or $\rho = \rho_0(1 - \gamma \Delta T)$ [valid for small ΔT]

Thermal Expansion of Liquid

Liquids do not have linear and superficial expansion but these only have volume expansion.

Liquid have two coefficient of volume expansion

1. Coefficient of apparent expansion,

   $$\gamma_a = \frac{\text{Apparent expansion in volume}}{\text{Initial volume} \times \Delta T} = \frac{(\Delta V)_a}{V \times \Delta T}$$

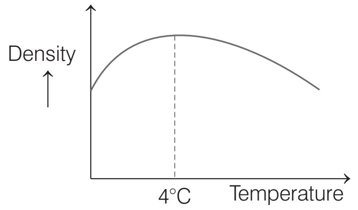
2. Coefficient of real expansion,

   $$\gamma_r = \frac{\text{Real expansion in volume}}{\text{Initial volume} \times \Delta T} = \frac{(\Delta V)_r}{V \times \Delta T}$$

Anomalous/Exceptional Behaviour of Water

Generally, density of liquids decreases with increase in temperature but for water as the temperature increases from 0 to 4°C, its density increases and as temperature increases beyond 4°C, the density decreases.

The variation in the density of water with temperature is shown in the figure given below.



Anomalous Behaviour of Water

Thermal Expansion of Gases

Gases have no definite shape, therefore, gases have only volume expansion.

1. The coefficient of volume expansion at constant pressure, $\alpha = \dfrac{\Delta V}{V_0} \times \dfrac{1}{\Delta T}$

   Final volume, $V' = V(1 + \alpha \Delta T)$

2. The coefficient of pressure expansion at constant volume, $\beta = \dfrac{\Delta p}{p} \times \dfrac{1}{\Delta T}$

   Final pressure, $p' = p(1 + \beta \Delta T)$.

Specific Heat

The quantity of heat required to raise the temperature of unit mass of a substance by 1°C is called specific heat.

Specific heat, $s = \frac{Q}{m \times \Delta T}$

The SI unit of specific heat is J kg^{-1} k^{-1}.

- Specific heat capacity can have any value from 0 to ∞. For some substances under particular situations, it can have negative values also.
- The product of mass of the body and specific heat is termed as **heat capacity**, $C = m \times s$.

Molar Heat Capacity

The amount of heat required to change the temperature of a unit mole of substance by 1°C is termed as its molar heat capacity,

$$C_m = \frac{Q}{\mu \Delta T}$$

Generally, for gases, two molar heat capacities are very common—molar heat capacity at constant pressure (C_p) and molar heat capacity at constant volume (C_V).

Water Equivalent of a Substance

Water equivalent of certain amount of substance is defined as the amount of water, which when replaced by the substance requires the same amount of heat for the same rise in temperature.

$$m_w = \frac{mS}{S_w},$$

where, m_w = water equivalent of substance whose mass is m,

S = specific heat capacity of substance

and S_w = specific heat capacity of water

Calorimetry

Calorimetry means measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body and provided no heat is allowed to escape to the surrounding. A device in which heat measurement can be made is called a calorimeter.

If temperature changes, heat exchanged is given by

$$Q = ms\Delta T$$

As temperature of the body increases, it means heat is taken by the body, otherwise given out by the body.

Latent Heat

In case of phase change, heat is consumed during melting and boiling while released during freezing and condensation.

The heat required to change the phase of a system is proportional to the mass of the system i.e.

$$Q \propto m$$

$$Q = mL$$

where, L is the latent heat, which is defined as the amount of heat required to change the phase of the unit mass of a substance at given temperature.

- In case of ice, the latent heat of fusion of ice is 80 cal/gm.
- In case of water, the latent heat of vapourisation is 539 cal/gm.

Heat Transfer

The heat can be transferred from one body to the other body, through the following modes

1. Conduction

The process of heat-transmission in which the particles of the body do not leave their position is called conduction.

Thermal Conductivity

The amount of heat transmitted through a conductor is given by $Q = \frac{KA\Delta T\, t}{l}$

where, A = area of cross-section,

ΔT = temperature difference $= T_2 - T_1$,

t = time elapsed,

K = thermal conductivity

and l = length of conductor

The rate of transmission of heat by conduction is given by

$$H = \frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{l}$$

The unit of thermal conductivity is $Wm^{-1}K^{-1}$.

Thermal Resistance

$$|H| = \left|\frac{\Delta Q}{\Delta t}\right| = \frac{KA}{l} \cdot \Delta T = \frac{\Delta T}{l / KA}$$

The term $\frac{l}{KA}$ is generally called the **thermal resistance** (R).

- Equation for rate of heat conduction can be written as

$$H = \frac{Q}{t} = \frac{\Delta T}{R_{\text{thermal}}}$$

It is equivalent/analysis to ohm's law which states that

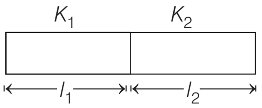
$$I = \frac{V}{R_{\text{(electrical)}}}$$

where, $H = \frac{Q}{t}$ is equivalent of electric current and called as heat, ΔT is equivalent of voltage (PD) and R_{thermal} is equivalent of $R_{\text{electrical}}$.

Combination of Metallic Rods

1. **Series Combination** In a series combination of two metal rods, equivalent thermal conductivity is given by

$$K_s = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}}$$

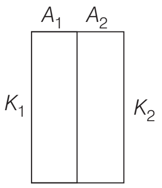


or $$K_s = \frac{2K_1K_2}{K_1 + K_2} \quad [\text{if } l_1 = l_2]$$

If temperature of the interface of the series combination be T, then

$$T = \frac{K_1T_1 + K_2T_2}{K_1 + K_2}$$

2. **Parallel Combinations** In a parallel combination of two metal rods, thermal conductivity is given by



$$K_p = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

or $$K_p = \frac{K_1 + K_2}{2} \quad [\text{if } A_1 = A_2]$$

Formation and Growth of Ice on a Lake

Time required for the thickness of the layer of ice to increase from d_1 to d_2 will be

$$t = \frac{\rho L_f}{2KT}(d_2^2 - d_1^2)$$

where, ρ = density of ice,

L_f = latent heat of fusion of ice

and K = thermal conductivity of ice

2. Convection

The process of heat-transmission in which the particles of the fluid move is called convection.

3. Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

Interaction of Radiation with Matter

When radiant energy Q is incident on a body, a part of it Q_a is absorbed, another part Q_r is reflected back and yet another part Q_t is transmitted such that

$$Q = Q_a + Q_r + Q_t$$

or $$\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = 1 \quad \text{or} \quad a + r + t = 1$$

where, $a = \frac{Q_a}{Q}$ = absorbing power or absorptance,

$r = \frac{Q_r}{Q}$ = reflecting power or reflectance

and $t = \frac{Q_t}{Q}$ = transmitting power or transmittance

Perfectly Black Body

A perfectly black body is the one which completely absorbs the radiations of all the wavelengths that are incident on it. Thus, absorbing power of a perfectly black body is 1 (i.e $a = 1$).

When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. e.g. temperature of the sun is very high (6000 k approx.) it emits all possible radiations. So, it is an example of black body.

- For perfectly black body, $a = 1, \; r = t = 0$
- For a perfect reflector, $a = t = 0, r = 1$
- For a perfect transmitter, $a = r = 0, t = 1$.

Emissive Power and Emissivity

Total **emissive power** of a given surface at a given temperature is defined as the total amount of radiant energy emitted per unit surface area per unit time by the body. SI unit of emissive power is Wm^{-2}.

Emissive power of a surface depends on the nature of the surface and its temperature.

Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the black body (e) to the total emissive power of perfectly black body (E) at that temperature

i.e. $$\varepsilon = \frac{e}{E}$$

A perfectly black body is also the perfect emitter i.e. it emits radiations of all possible wavelengths at that temperature.

Absorptive Power

The ratio of the radiant energy absorbed by a surface in a given time to the total radiant energy incident on the surface in the same time, is called the absorptive power.

Kirchhoff 's Law of Radiation

Kirchhoff's law of radiation states that the ratio of emissive power to absorptive power of a body, is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

Mathematically, $\frac{e_1}{a_1} = \frac{e_2}{a_2} = \ldots = E = 1 \quad \text{or} \quad e = a$

Kirchhoff's law implies that 'a good absorber is a good emitter (or radiator) too'.

Fraunhoffer's lines (dark lines observed in solar spectrum) can be easily explained on the basis of Kirchhoff's laws.

Stefan's Law

According to the Stefan's law, the emissive power of a perfectly black body (energy emitted by black body per unit surface area per unit time) is directly proportional to the fourth power of its absolute temperature.

Mathematically,

$$E \propto T^4 \text{ or } E = \sigma T^4$$

where, σ is a constant known as the Stefan's constant and its value is 5.67×10^{-8} $\text{Wm}^{-2}\,\text{K}^{-4}$.

For a body, whose emissivity is ε, Stefan's law is modified as, $e = \varepsilon\sigma T^4$.

The total radiant energy Q emitted by a body of surface area A in time t, is given by

$$Q = Ate = At\varepsilon\sigma T^4.$$

The radiant power (P), i.e. energy radiated by a body per unit time is given by

$$P = \frac{Q}{t} = A\varepsilon\sigma T^4$$

If a body at temperature T is surrounded by another body at temperature T_0 (where, $T_0 < T$), then Stefan's law is modified as,

$$E = \sigma (T^4 - T_0^4) \qquad \text{[black body]}$$

and

$$e = \varepsilon\sigma (T^4 - T_0^4) \qquad \text{[any body]}$$

Solar Constant

The amount of heat received from the sun by one square centimeter area of a surface placed normally to the sun rays at mean distance of the earth from the sun is known as solar constant. It is denoted by S.

$$S = \left(\frac{r}{R}\right)^2 \sigma T^4$$

where, r is the radius of sun and R is the mean earth's distance from sun value of solar constant $S = 1.937$ cal/cm^2/min.

Newton's Law of Cooling

According to the Newton's law of cooling, rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings. The law holds good only for small temperature difference.

Mathematically, $-\frac{dT}{dt} \propto (T - T_0)$

or $$-\frac{dT}{dt} = k(T - T_0)$$

where, k is a constant.

If a body cools by radiation through a small temperature difference from T_1 to T_2 in a short time t when the surrounding temperature is T_0, then

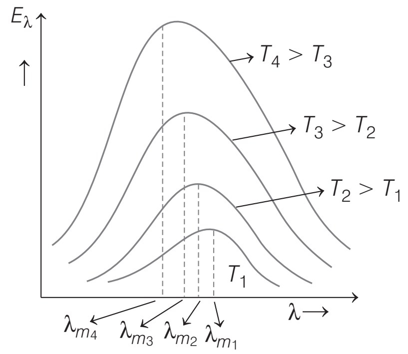
$$-\frac{dT}{dt} \simeq \frac{T_1 - T_2}{t} \quad \text{and } T = \frac{T_1 + T_2}{2}$$

The Newton's law of cooling becomes

$$\left[\frac{T_1 - T_2}{t}\right] = k\left[\frac{T_1 + T_2}{2} - T_0\right].$$

Black Body Spectrum

The black body spectrum is a continuous spectrum as shown in the figure. At a given temperature, initially the intensity of thermal radiation increases with an increase in wavelength and reaches a maximum value at a particular wavelength λ_m. On increasing the wavelength beyond λ_m, the intensity of radiation E_λ starts decreasing.



The total area under E_λ-λ curve gives the total intensity of radiation at that temperature.

The area, in accordance with the Stefan's law of radiation, is directly proportional to the fourth power of the temperature.

Wien's Displacement Law

According to Wien's law, the product of wavelength corresponding to maximum intensity of radiation and temperature of body is constant i.e. $\lambda_m T$ = constant = b, where b is known as the Wien's constant and its value is 2.89×10^{-3} mK.

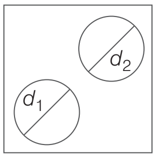
Green House Effect

The carbon dioxide concentration in the atmosphere has increased which may be attributed to the increase in the temperature of atmosphere. This effect is known as the green house effect.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The metal sheet as shown in figure with two holes cut-off unequal diameters d_1 and d_2 $(d_1 > d_2)$. If the sheet is heated,



(a) both d_1 and d_2 will decrease
(b) both d_1 and d_2 will increase
(c) d_1 will increase, d_2 will decrease
(d) d_1 will decrease, d_2 will increase

2 What will be the force developed in steel rod of cross-sectional area $150\,mm^2$, which is fixed between two fixed points, if temperature is increased by 20°C?
(Assume, $\alpha = 10^{-5}$/°C and $Y = 200 \times 10^{11} N/m^2$)

(a) 200 kN (b) 400 kN (c) 600 kN (d) 800 kN

3 The value of coefficient of volume expansion of glycerin is $5 \times 10^{-4}\ K^{-1}$. The fractional change in the density of glycerin for a rise of 40°C in its temperature is ➜ CBSE AIPMT 2015

(a) 0.015 (b) 0.020
(c) 0.025 (d) 0.010

4 A clock with a metal pendulum beating seconds keeps correct time at 0°C. If it loses 12.5 s a day at 25°C, the coefficient of linear expansion of metal pendulum is

(a) $\frac{1}{86400}$ /°C (b) $\frac{1}{43200}$ /°C (c) $\frac{1}{14400}$ /°C (d) $\frac{1}{28800}$ /°C

5 In similar calorimeters, equal volumes of water and alcohol when poured take 100 s and 74 s respectively to cool from 50°C to 40°C. If the thermal capacity of each calorimeter is numerically equal to volume of either liquid, then the specific heat capacity of alcohol is
(Given, relative density of alcohol as 0.8 and specific heat capacity of water as 1 cal/g/°C)

(a) 0.8 cal/g°C (b) 0.6 cal/g°C
(c) 0.9 cal/g°C (d) 1 cal/g°C

6 Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is at 100°C, while the other one is at 0°C. If the two bodies are brought into contact, then assuming no heat loss, the final common temperature is ➜ NEET 2016

(a) 50°C (b) more than 50°C
(c) less than 50°C but greater than 0°C
(d) 0°C

7 2 g of steam condenses, when passed through 40 g of water initially at 25°C. The condensation of steam raises the temperature of water to 54.3°C. What is the latent heat of steam?

(a) 540 cal/g (b) 536 cal/g
(c) 270 cal/g (d) 480 cal/g

8 Steam at 100°C is passed into 20 g of water at 10°C. When water acquires temperature of 80°C, the mass of water present will be
(Take, specific heat of water = 1 cal g^{-1} ° C^{-1} and latent heat of steam = 540 cal g^{-1}) ➜ CBSE AIPMT 2014

(a) 24 g (b) 31.5 g (c) 42.5 g (d) 22.5 g

9 A piece of ice falls from a height h so that it melts completely. Only one-quarter of the heat produced is absorbed by the ice and all energy of ice gets converted into heat during its fall. The value of h is [Latent heat of ice is 3.4×10^5 J/kg and $g = 10\,N/kg$] ➜ NEET 2016

(a) 544 km (b) 136 km (c) 68 km (d) 34 km

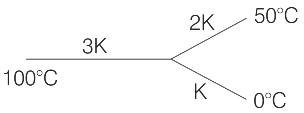
10 In a steady state, the temperature at the ends A and B of 20 cm long rod AB are 100°C and 0°C, respectively. The temperature of a point 9 cm from A is

(a) 45°C (b) 60°C (c) 55°C (d) 65°C

11 The heat is flowing through a rod of length 50 cm and area of cross-section 5 cm^2. Its ends are respectively at 25°C and 125°C. The coefficient of thermal conductivity of the material of the rod is 0.092 kcal/ms. The temperature gradient in the rod is

(a) 2°C/cm (b) 2°C/m (c) 20°C/cm (d) 20°C/m

12 Three rods of same dimensions have thermal conductivities 3 K, 2 K and K. They are arranged as shown, with their ends at 100°C, 50°C and 0°C. The temperature of their junction is



(a) 75°C (b) $\frac{200}{3}$°C (c) 40°C (d) $\frac{100}{3}$°C

13 The two ends of a metal rod are maintained at temperatures 100°C and 110°C. The rate of heat flow in the rod is found to be 4.0 J/s. If the ends are maintained at temperatures 200°C and 210°C, the rate of heat flow will be ➜ CBSE AIPMT 2015

(a) 44.0 J/s (b) 16.8 J/s (c) 8.0 J/s (d) 4.0 J/s

14 The ratio of the emissive power to the absorptive power of all substances for the particular wavelength is the same at given temperature. The ratio is known as

(a) the emissive power of a perfectly black body
(b) the emissive power of any type of body
(c) the Stefan's constant
(d) the Wien's constant

15 A black body is at 727°C. It emits energy at a rate which is proportional to

(a) $(727)^2$ (b) $(1000)^4$ (c) $(1000)^2$ (d) $(727)^4$

16 A sphere has a surface area of $1.0 m^2$ and a temperature of 400 K and the power radiated from it is 150 W. Assuming the sphere is a black body radiator, the power in kilowatt radiated when the area expands to $2.0 m^2$ and the temperature changes to 800 K is

(a) 6.2 (b) 9.6
(c) 4.8 (d) 16

17 Two spheres of the same material have radii 1 m and 4 m, temperature 4000K and 2000K, respectively. Then, the ratio of energy radiated per second by the first sphere as compared to that by the second is

(a) 4 : 1 (b) 2 : 1
(c) 1 : 1 (d) 1 : 4

18 If a black body radiates 10 cal s^{-1} at 227°C, it will radiate at 727°C

(a) 10 cal s^{-1} (b) 80 cal s^{-1}
(c) 160 cal s^{-1} (d) None of these

19 The surface temperature of the sun is T K and the solar constant for a plate is S. The sun subtends an angle θ at the planet. Then,

(a) $S \propto T^4$ (b) $S \propto T^2$
(c) $S \propto \theta^2$ (d) $S \propto \theta$

20 If the radius of a star is R and it acts as a black body, what would be the temperature of the star in which the rate of energy production is Q? ➜ CBSE AIPMT 2012

(a) $\frac{Q}{4\pi R^2 \sigma}$ (b) $\left(\frac{Q}{4\pi R^2 \sigma}\right)^{1/2}$
(c) $\left(\frac{4\pi R^2 Q}{\sigma}\right)^{1/4}$ (d) $\left(\frac{Q}{4\pi R^2 \sigma}\right)^{1/4}$

where, σ stands for Stefan's constant.

21 The total radiant energy per unit area, normal to the direction of incidence, received at a distance R from the centre of a star of radius r, whose outer surface radiates as a black body at a temperature T K is given by ➜ CBSE AIPMT 2011

(a) $\frac{\sigma r^2 T^4}{R^2}$ (b) $\frac{\sigma r^2 T^4}{4\pi r^2}$
(c) $\frac{\sigma r^2 T^4}{r^4}$ (d) $\frac{4\pi\sigma r^2 T^4}{R^2}$

22 A sphere, a cube and a thin circular plate, all of same material and of same mass are initially heated to same high temperature

(a) plate will cool fastest and cube the slowest
(b) sphere will cool fastest and cube the slowest
(c) plate will cool fastest and sphere the slowest
(d) cube will cool fastest and plate the slowest

23 A black body at 227°C radiates heat at the rate of 7 cal-$cm^{-2}s^{-1}$. At a temperature of 727°C, the rate of heat radiated in the same units will be ➜ CBSE AIPMT 2009

(a) 60 (b) 50 (c) 112 (d) 80

24 The initial temperature of a body is 80°C. If its temperature falls to 64°C in 5 minutes and in 10 minutes to 52°C, then the temperature of surrounding will be

(a) 26°C (b) 49.6°C (c) 35°C (d) 42°C

25 A body cools from a temperature $3T$ to $2T$ in 10 minutes. The room temperature is T. Assume that Newton's law of cooling is applicable. The temperature of the body at the end of next 10 minutes will be ➜ NEET 2016

(a) $\frac{7}{4}T$ (b) $\frac{3}{2}T$
(c) $\frac{4}{3}T$ (d) T

26 The two ends of a rod of length L and a uniform cross-sectional area A are kept at two temperatures T_1 and $T_2 (T_1 > T_2)$. The rate of heat transfer, $\frac{dQ}{dt}$, through the rod in a steady state is given by ➜ CBSE AIPMT 2009

(a) $\frac{dQ}{dt} = \frac{KL(T_1 - T_2)}{A}$ (b) $\frac{dQ}{dt} = \frac{K(T_1 - T_2)}{LA}$
(c) $\frac{dQ}{dt} = KLA(T_1 - T_2)$ (d) $\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L}$

27 There are two identical vessel filled with equal amount of ice. The vessels are of different metals. If the ice melts in the two vessels in 20 min and 35 min respectively, the ratio of the coefficient of thermal conductivity of the two metals is

(a) 4 : 7 (b) 7 : 4
(c) 16 : 49 (d) 49 : 16

28 A black body at a temperature of 2600 K has the wavelength corresponding to maximum emission 1200 Å assuming the moon to be perfectly black body. The temperature of the moon, if the wavelength corresponding to maximum emission is 5000 Å, is

(a) 7800 K (b) 624 K
(c) 5240 K (d) 3640 K

29 Certain quantity of water cools from 70°C to 60°C in the first 5 minutes and to 54°C in the next 5 minutes. The temperature of the surroundings is ➜ CBSE AIPMT 2014

(a) 45°C (b) 20°C
(c) 42°C (d) 10°C

30 If a body coated black at 600 K surrounded by atmosphere at 300 K has cooling rate r_0, the same body at 900 K, surrounded by the same atmosphere, will have cooling rate equal to

(a) $\frac{16}{3} r_0$ (b) $\frac{8}{16} r_0$ (c) $16 r_0$ (d) $4 r_0$

31 The temperature of sun is 5500 K and it emits maximum intensity radiation in the yellow region (5.5×10^{-7} m). The maximum radiation from a furnance occurs at wavelength 11×10^{-7} m. The temperature of furnace is

(a) 1125 K (b) 2750 K
(c) 5500 K (d) 11000 K

32 A black body is at a temperature of 5760 K. The energy of radiation emitted by the body at wavelength 250 nm is U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3. Wien's constant, $b = 2.88 \times 10^6$ nmK. Which of the following is correct ? ➜ NEET 2016

(a) $U_3 = 0$ (b) $U_1 > U_2$
(c) $U_2 > U_1$ (d) $U_1 = 0$

33 On observing light from three different stars P, Q and R, it was found that intensity of violet colour is maximum in the spectrum of P, the intensity of green colour is maximum in the spectrum of R and the intensity of red colour is maximum in the spectrum of Q. If T_P, T_Q and T_R are the respective absolute temperatures of P, Q and R, then it can be concluded from the above observations that ➜ CBSE AIPMT 2015

(a) $T_P > T_Q > T_R$ (b) $T_P > T_R > T_Q$
(c) $T_P < T_R < T_Q$ (d) $T_P < T_Q < T_R$

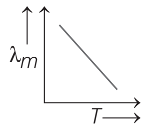
34 If λ_m denotes the wavelength at which the radiative emission from a black body at a temperature T K is maximum, then

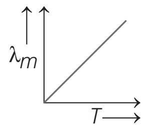
(a) $\lambda_m \propto T^4$ (b) λ_m is independent of T
(c) $\lambda_m \propto T$ (d) $\lambda_m \propto T^{-1}$

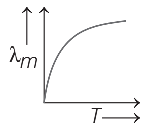
35 A piece of iron is heated in a flame. If first becomes dull red then becomes reddish yellow and finally turns to white hot. The correct explanation for the above observations is possible by using ➜ NEET 2013

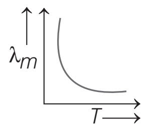
(a) Stefan's law (b) Wien's displacement law
(c) Kirchhoff's law (d) Newton's law of cooling

36 Which of the following is the λ_m -Tgraph for a perfectly black body?

(a)


(b)


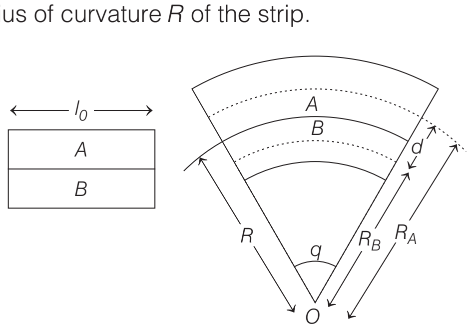
(c)


(d)


DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 At temperature T_0, two metal strips of length l_0 and thickness d, is bolted, so that their ends coincide. The upper strip is made up of metal A and have coefficient of expansion α_A and lower strip is made up of metal B with coefficient of expansion α_B ($\alpha_A > \alpha_B$). When temperature of their blastic strip is increased from T_0 to $(T_0 + \Delta T)$, one strip becomes longer than the other and blastic strip is bend in the form of a circle as shown in figure. Calculate the radius of curvature R of the strip.



(a) $R = \frac{[2 + (\alpha_A + \alpha_B)\,\Delta T]d}{2\,(\alpha_A - \alpha_B)\,\Delta T}$ (b) $R = \frac{[2 - (\alpha_A + \alpha_B)\,\Delta T]d}{2\,(\alpha_A - \alpha_B)\,\Delta T}$

(c) $R = \frac{[2 + (\alpha_A - \alpha_B)\,\Delta T]d}{2\,(\alpha_A - \alpha_B)\,\Delta T}$ (d) $R = \frac{[2 - (\alpha_A - \alpha_B)\,\Delta T]d}{2\,(\alpha_A - \alpha_B)\,\Delta T}$

2 The specific heat of a substance at temperature t°C is $s = at^2 + bt + c$. The amount of heat required to raise the temperature of m kg of the substance from 0°C to t_0°C is

(a) $\frac{mt_0^3 a}{3} + \frac{bt_0^2}{2} + ct_0$ (b) $\frac{mt_0^3 a}{3} + \frac{mbt_0^2}{2} + mct_0$

(c) $\frac{mt_0^3 a}{3} + \frac{mbt_0^2}{2}$ (d) None of these

3 A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be ➜ NEET 2017

(a) 225 (b) 450
(c) 1000 (d) 1800

4 A wall has two layers A and B, each made of different materials. Both the layers have the same thickness. The thermal conductivity for A is twice that of B and under steady condition, the temperature difference across the wall is 36°C. The temperature difference across the layer A is

(a) 6°C (b) 12°C (c) 24°C (d) 18°C

5 Ice starts forming in lake with water at 0°C and when the atmospheric temperature is –10°C. If the time taken for 1 cm of ice be 7 h, then the time taken for the thickness of ice to change from 1 cm to 2 cm, is

(a) 7 h (b) 14 h
(c) less than 7 h (d) more than 7 h

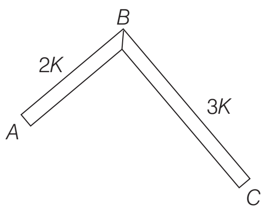
6 A wall has two layers A and B each made of different materials. The layer A is 10 cm thick and B is 20 cm thick. The thermal conductivity of A is thrice that of B. Under thermal equilibrium, temperature difference across the wall is 35°C. The difference of temperature across the layer A is

(a) 20°C (b) 10°C (c) 15°C (d) 5°C

7 A cylindrical metallic rod in thermal contact with two reservoirs of heat at its two ends conducts an amount of heat Q in time t. The metallic rod is melted and the material is formed into a rod of half the radius of the original rod. What is the amount of heat conducted by the new rod when placed in thermal contact with the two reservoirs in time t? ➔ CBSE AIPMT 2010

(a) $Q/4$ (b) $Q/16$
(c) $2Q$ (d) $Q/2$

8 In the figure, ABC is a conducting rod whose lateral surfaces are insulated. The length of the section AB is one-half of that of BC and the respective thermal conductivities of the two sections are as given in the figure. If the ends A and C are maintained at 0°C and 70°C respectively, the temperature of junction B in the steady state is



(a) 30°C (b) 40°C (c) 50°C (d) 60°C

9 Two identical square rods of metal are welded end to end as shown in Fig. (i), 20 cal of heat flows through it in 4 min. If the rods are welded as shown in Fig. (ii), the same amount of heat will flow through the rods in

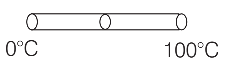


Fig. (i)

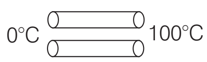


Fig. (ii)

(a) 1 min (b) 2 min (c) 4 min (d) 16 min

10 Assuming the sun to have a spherical outer surface of radius r, radiating like a black body at temperature t°C, the power received by a unit surface, (normal to the incident rays) at a distance R from the centre of the sun is (where, σ is the Stefan's constant.)

(a) $\frac{4\pi r^2 t^4}{R^2}$ (b) $\frac{r^2 \sigma (t+273)^4}{4\pi R^2}$

(c) $\frac{16\pi^2 r^2 \sigma t^4}{R^2}$ (d) $\frac{r^2 \sigma (t+273)^4}{R^2}$

11 A black body calorimeter filled with hot water cools from 60°C to 50°C in 4 min and 40°C to 30°C in 8 min. The approximate temperature of surrounding is

(a) 10°C (b) 15°C (c) 20°C (d) 25°C

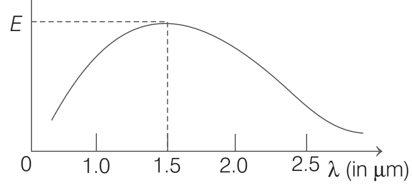
12 The power radiated by a black body is P and it radiates maximum energy at wavelength, λ_0. If the temperature of the black body is now changed, so that it radiates maximum energy at wavelength $\frac{3}{4}\lambda_0$, the power radiated by it becomes nP. The value of n is ➔ NEET 2018

(a) $\frac{256}{81}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{81}{256}$

13 A black body at 1227°C emits radiations with maximum intensity at a wavelength of 5000 Å. If the temperature of the body is increased by 1000°C, the maximum intensity will be observed at

(a) 4000 Å (b) 5000 Å (c) 6000 Å (d) 3000 Å

14 In the figure, the distribution of energy density of the radiation emitted by a black body at a given temperature is shown. The possible temperature of the black body at $\lambda_m = 1.5\,\mu$m is



(a) 1500 K (b) 2000 K (c) 2500 K (d) 3000 K

15 A non-conducting body floats in a liquid at 20°C with $\frac{2}{3}$ of its volume immersed in the liquid. When liquid temperature is increased to 100°C, $\frac{3}{4}$ of body's volume is immersed in the liquid. Then, the coefficient of real expansion of the liquid is (neglecting the expansion of container of the liquid)

(a) $15.6 \times 10^{-4}\,°C^{-1}$
(b) $156 \times 10^{-4}\,°C^{-1}$
(c) $1.56 \times 10^{-4}\,°C^{-1}$
(d) $0.156 \times 10^{-4}\,°C^{-1}$

16 Two slabs A and B of different materials but of the same thickness are joined end to form a composite slab. The thermal conductivities of A and B are K_1 and K_2 respectively. A steady temperature difference of 12°C is maintained across the composite slab. If $K_1 = \frac{K_2}{2}$, the temperature difference across slabs A is

(a) 4°C (b) 6°C
(c) 8°C (d) 10°C

17 An experiment takes 10 min to raise temperature of water from 0°C and 100°C and another 55 min to convert it totally into steam by a stabilised heater. The latent heat of vaporisation comes out to be

(a) 530 cal/g
(b) 540 cal/g
(c) 550 cal/g
(d) 560 cal/g

ANSWERS

SESSION 1	**1** (b)	**2** (c)	**3** (b)	**4** (a)	**5** (b)	**6** (b)	**7** (a)	**8** (d)	**9** (b)	**10** (c)
	11 (a)	**12** (b)	**13** (d)	**14** (a)	**15** (b)	**16** (c)	**17** (c)	**18** (c)	**19** (a)	**20** (d)
	21 (a)	**22** (c)	**23** (c)	**24** (b)	**25** (b)	**26** (d)	**27** (b)	**28** (b)	**29** (a)	**30** (a)
	31 (b)	**32** (c)	**33** (b)	**34** (d)	**35** (b)	**36** (d)				
SESSION 2	**1** (a)	**2** (b)	**3** (d)	**4** (b)	**5** (d)	**6** (d)	**7** (b)	**8** (a)	**9** (a)	**10** (d)
	11 (b)	**12** (a)	**13** (d)	**14** (b)	**15** (a)	**16** (c)	**17** (c)			

Hints and Explanations

SESSION 1

1 When a sheet having positive coefficient of expansion is heated, diameters of holes present in the sheet will increase.

2 Since, the support is fixed, so its length won't change due to increase in temperature. Instead of expansion, the rod shall undergo compressive stress σ, such that $\Delta l_{\text{expansion}} + \Delta l_{\text{stress}} = 0$

$$\Rightarrow \quad \alpha\, l\,.\,\Delta T - \frac{\sigma l}{\lambda} = 0$$

$$\Rightarrow \quad \sigma = \alpha\,\lambda\,.\,\Delta T$$

$$\Rightarrow \quad \sigma = 10^{-5} \times 200 \times 10^{11} \times 20$$

$$\therefore \text{ Force } = \sigma\, A = 150 \times 10^{-6} \times 10^{-5} \times 200 \times 10^{11} \times 20$$

$$= 600000 \text{ N} = 600 \text{ kN}$$

3 Given, the value of coefficient of volume expansion of glycerin is $5 \times 10^{-4}\ \text{K}^{-1}$.

As, orginal density of glycerin,

$$\rho = \rho_0\,(1 + Y\Delta T)$$

$$\Rightarrow \quad \rho - \rho_0 = \rho_0 Y\Delta T$$

Thus, fractional change in the density of glycerine for a rise of 40°C in its temperature,

$$\frac{\rho - \rho_0}{\rho_0} = Y\Delta T = 5 \times 10^{-4} \times 40$$

$$= 200 \times 10^{-4} = 0.020$$

4 Number of second lost in a day

$$\Delta t = \frac{1}{2}\,\alpha\,\Delta\theta \times 86400$$

The coefficient of linear expansion of metal pendulum,

$$\alpha = \frac{2\Delta t}{\Delta\theta \times 86400} = \frac{2 \times 12.5}{25 \times 86400}$$

$$\alpha = \frac{1}{86400}\,/°\text{C}$$

5 Let, V be volume of either liquid, mass of water $= V \times 1\ g$

Mass of alcohol $= V \times 0.8 = 0.8\,Vg$

Rate of cooling of the water calorimeter

$$= \frac{1}{100}\,[V \times (50° - 40°) + V \times 1 \times (50° - 40°)]$$

$$= \frac{1}{5}\,V \text{ cal/s}$$

Rate of cooling of alcohol calorimeter

$$= \frac{1}{74}\,[V \times (50° - 40°) + 0.8V \times s\,(50° - 40°)]$$

$$= \frac{1}{74}\,(10V + 8Vs) \text{ cal/s}$$

As, rate of cooling of both is same

$$\frac{V}{5} = \frac{1}{74}\,(10V + 8Vs)$$

$$\Rightarrow \quad s = 0.6 \text{ cal/g°C}$$

6 Heat lost by Ist body = heat gained by IInd body. Body at 100°C temperature has greater heat capacity than body at 0°C so final temperature will be closer to 100°C. So, $T_c > 50$°C.

7 Let L be the latent heat and using principle of calorimetry,

$$2L + 2(100 - 54.3) = 40 \times (54.3 - 25)$$

$$L = 540.3 \text{ cal/g}$$

8 Heat given by water = Heat lost by steam

$$20 \times 1 \times (80 - 10) = m \times 540 + m \times 1 \times (100 - 80)$$

$$\Rightarrow \quad 1400 = 560m$$

$$\Rightarrow \quad m = 2.5 \text{ g}$$

Total mass of water $= 20 + 2.5 = 22.5$ g

9 According to question as conservation of energy, energy gained by the ice during its fall from height h is given by

$$E = mgh$$

As given, only one quarter of its energy is absorbed by the ice.

So, $\frac{mgh}{4} = mL_f \Rightarrow h = \frac{mL_f \times 4}{mg}$

$$= \frac{L_f \times 4}{g} = \frac{3.4 \times 10^5 \times 4}{10}$$

$$= 13.6 \times 10^4$$

$$= 136000 \text{ m} = 136 \text{ km}$$

10 $H = \frac{100-0}{R_{AB}}$ = Heat current $= \frac{100}{R_{AB}}$

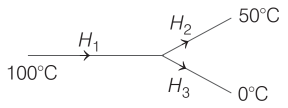
$$100 - T_C = HR_{AC} = \frac{100}{R_{AB}} R_{AC} = \frac{100 \times 9}{20} = 45$$

$$T_C = 55°C$$

11 Temperature gradient

$$\frac{dT}{dx} = \frac{(125-25)}{50\text{cm}} = 2°\text{C/cm}$$

12 Suppose T be the temperature of junction. H_1, H_2 and H_3 the heat currents. Then,



$$H_1 = H_2 + H_3$$

or $$\frac{100 - T}{\left(\frac{l}{3KA}\right)} = \frac{T-50}{\left(\frac{l}{2KA}\right)} + \frac{T-0}{\left(\frac{l}{KA}\right)}$$

or $3(100 - T) = 2(T - 50) + T$

$\Rightarrow$ $T = \frac{200}{3}$ °C

13 In both cases the temperature difference between the ends of the rod is 10°C.

$\therefore$ Rate of heat flow is also 4 J/S in the second case.

14 Kirchhoff's law of radiation states that the ratio of emissive power to absorptive power of a body is same for all surface at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

15 According to Stefan's law,

$E \propto T^4$

or $E = \sigma T^4$,

Here, $E \propto (727 + 273)^4$

[σ = Stefan's constant]

$\Rightarrow$ $E \propto (1000)^4$

16 $P \propto AT^4$

Area and temperature both are double. Hence, power will become $(2)^5$ or 32 times.

$\therefore P' = 32P = \frac{32 \times 150}{1000}$ kW

$= 4.8$ kW

17 $$\frac{P_1}{P_2} = \frac{A_1 T_1^4}{A_2 T_2^4} = \frac{4\pi(1)^2(4000)^4}{4\pi(4)^2(2000)^4} = \frac{1}{1}$$

18 $E \propto T^4, T_1 = 227°C = 500\text{K}$

and $T_2 = 727°C = 1000\text{K}$

Hence, $T_2 = 2T_1$

$\therefore$ $E_2 = (2)^4 E_1$

$= 16E_1 = 160 \text{ cal s}^{-1}$

19 Let radius of the sun $= R$

Distance of the earth from the sun $= d$

Power radiated from the sun

$$P = (4\pi R^2)\sigma T^4$$

Power received/area $= S = \frac{P}{4\pi d^2}$

$$= 4\pi R^2 \sigma \frac{T^4}{4\pi d^2}$$

$$= \sigma T^4 \frac{R^2}{d^2}$$

$$= \frac{1}{4}\sigma T^4 \left(\frac{2R}{d}\right)^2$$

Angle subtended by sun at the earth $\alpha = \frac{2R}{d}$.

$$S = \text{constant} \times T^4 \times \alpha^2$$

$\therefore$ $S \propto T^4$

20 From Stefan's law,

$$E = \sigma T^4$$

So, the rate of energy production,

$$Q = E \times A$$

$$Q = \sigma T^4 \times 4\pi R^2$$

Temperature of star, $T = \left(\frac{Q}{4\pi R^2 \sigma}\right)^{1/4}$

21 If r is the radius of the star and T is its temperature, then the energy emitted by the star per second through radiation in accordance with Stefan's law will be given by

$$A \sigma T^4 = 4\pi r^2 \sigma T^4$$

In reaching a distance R, this energy will spread over a sphere of radius R, so the intensity of radiation will be given by

$$S = \frac{P}{4\pi R^2} = \frac{4\pi r^2 \sigma T^4}{4\pi R^2} = \frac{r^2 \sigma T^4}{R^2}$$

22 Rate of loss of heat from a body is directly proportional to the surface area of the body for a given mass of material, the surface area of circular plate is maximum and of sphere is least. Hence, plate will cool the fastest and sphere will cool the slowest.

23 According to Stefan's law

$E = \sigma T^4$

σ = Stefan's constant

T = temperature

Given, $T_1 = 227°C$ and $T_2 = 727°C$

$E_1 = 7 \text{ cal cm}^{-2} \text{ s}^{-1}$

$$\frac{E_1}{E_2} = \left[\frac{T_1}{T_2}\right]^4$$

$\Rightarrow$ $$E_2 = 7\left[\frac{273 + 727}{273 + 227}\right]^4 = \left[\frac{1000}{500}\right]^4 \times 7$$

$= 112 \text{ cal-cm}^2\text{s}^{-1}$

24 According to Newton's law,

$$\frac{T_1 - T_2}{t} = k\left[\frac{T_1 + T_2}{2} - T_0\right]$$

Initially, $\left(\frac{80-64}{5}\right) = k\left(\frac{80+64}{2} - T_0\right)$

$\Rightarrow$ $3.2 = k(72 - T_0)$ ...(i)

Finally,

$$\frac{64-52}{10} = k\left(\frac{64+52}{2} - T_0\right)$$

$1.2 = k(58 - T_0)$ ...(ii)

On solving Eqs. (i) and (ii), we get

$T_0 = 49.6°C$

25 According to Newton's law of cooling,

$$\Delta T = \Delta T_0 \, e^{-\lambda t}$$

$\Rightarrow$ $3T - 2T = (3T - T)e^{-\lambda \times 10}$ ...(i)

Again for next 10 minutes

$T' - T = (2T) \times e^{-\lambda(20)}$ ...(ii)

From Eqs. (i) and (ii), we get

$$T' - T = (2T)(e^{-\lambda \times 10})^2$$

$$= (2T)\left(\frac{1}{2}\right)^2 = \frac{T}{2}$$

$\therefore$ $T' = T + \frac{T}{2} = \frac{3T}{2}$

26 For a rod of length L and area of cross-section A whose faces are maintained at temperatures T_1 and T_2 respectively.

Then in steady state the rate of heat flowing from one face to the other face in time t is given by

$$\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L}$$

27 $$Q = \frac{kA(T_1 - T_2)t}{l}$$

$k_1 t_1 = k_2 t_2$ = constant

$$\frac{k_1}{k_2} = \frac{t_1}{t_2} = \frac{35}{20} = \frac{7}{4}$$

28 From Wien's law,

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$T_2 = \frac{\lambda_1 T_1}{\lambda_2} = \frac{1200 \times 2600}{5000}$$

$$T_2 = 624 \text{ K}$$

29 Newtons law of cooling

$$\frac{T_1 - T_2}{\Delta t} = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

First $\Rightarrow \dfrac{70-60}{5} = K\,[65 - T_0]$

$\Rightarrow \quad 2 = K\,[65 - T_0] \quad$ ...(i)

Next $\Rightarrow \dfrac{60-54}{5} = K\,[57 - T_0] \quad$ ...(ii)

On dividing Eqs. (i) and (ii), we get

$$\frac{5}{3} = \frac{65 - T_0}{57 - T_0}$$

$\Rightarrow \quad 285 - 5T_0 = 195 - 3T_0$

$\Rightarrow \quad 2T_0 = 90$

$T_0 = 45^\circ \text{C}$

30 Cooling rate $\propto T^4 - T_0^4$

$$r = \left\{\frac{(900)^4 - (300)^4}{(600)^4 - (300)^4}\right\} r_0$$

$$= \frac{16}{3} r_0$$

31 $\lambda_{m_1} T_1 = \lambda_{m_2} T_2$

$5.5 \times 10^{-7} \times 5500 = 11 \times 10^{-7}\,\text{T}$

$T = 550 \times 5\,\text{K} = 2750\,\text{K}$

32 Given, temperature, $T_1 = 5760\,\text{K}$

Since, it is given that energy of radiation emitted by the body at wavelength 250 nm in U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3.

$\because$ According to Wien's law, we get

$\lambda_m T = b$

where, b = Wien's constant

$= 2.88 \times 10^6$ nmK

$\Rightarrow \lambda_m = \dfrac{b}{T} \Rightarrow \lambda_m = \dfrac{2.88 \times 10^6 \text{ nmK}}{5760 \text{ K}}$

$\Rightarrow \quad \lambda_m = 500$ nm

$\because \lambda_m$ = wavelength corresponding to maximum energy, so, $U_2 > U_1$.

33 We know from Wien's displacement law,

$\lambda_m T$ = constant

So, $\quad T \propto \dfrac{1}{\lambda_m}$

As, $\quad \lambda_r > \lambda_g > \lambda_v$

So, $\quad T_r < T_g < T_v$

Given, $P \to v_{\max}, Q \to r_{\max}, R \to g_{\max}$

Hence, $\quad T_Q < T_R < T_P$

34 According to Wien's displacement law, the wavelength (λ_m) of maximum intensity of emission of black body radiation is inversely proportional to absolute temperature (T) of the black body. Therefore, Wien's law is

$\lambda_m T$ = constant

or $\quad \lambda_m \propto T^{-1}$

35 The equation of Wien's displacement law, i.e.

$\lambda_m T$ = constant

36 For perfectly black body,

$$\lambda_m \propto \frac{1}{T}$$

So, graph is rectangular hyperbola as shown in option (d).

SESSION 2

1 At temperature $T_0 + \Delta T$, increase in length

$L_A = l_0(1 + \alpha_A \Delta T) \quad$ ...(i)

$L_B = l_0(1 + \alpha_B \Delta T) \quad$ ...(ii)

Let R_A and R_B be the radius of strips,

$L_A = \theta R_A \quad$ ...(iii)

$L_B = \theta R_B \quad$ ...(iv)

as θ is the common angle.

From Eqs. (i) and (ii), we get

$L_A - L_B = \theta\,(R_A - R_B)$

$\Rightarrow \quad \theta = \dfrac{L_A - L_B}{R_A - R_B}$

$= \dfrac{l_0(\alpha_A - \alpha_B)\,\Delta T}{d} \quad$ ...(v)

Similarly from Eqs. (i), (ii), (iii), and (iv), we get

$R_A + R_B = \dfrac{2l_0 + l_0\,(\alpha_A + \alpha_B)\,\Delta T}{\theta} \quad$ ...(vi)

Mean radius, $R = \dfrac{R_A + R_B}{2}$

$= \dfrac{2l_0 + l_0(\alpha_A + \alpha_B)\,\Delta T}{2\theta} \quad$ ...(vii)

Putting the value of θ in Eq. (iii), we get

$$R = \frac{[2l_0 + l_0(\alpha_A + \alpha_B)\,\Delta T]}{2\left[l_0(\alpha_A - \alpha_B)\left(\dfrac{\Delta T}{d}\right)\right]}$$

$\Rightarrow \quad R = \dfrac{[2 + (\alpha_A + \alpha_B)\,\Delta T]\,d}{2\,(\alpha_A - \alpha_B)\,\Delta T}$

2 $\Delta H = \int ms\,dt = m\int_0^{t_0} (at^2 + bt + c)\,dt$

$= m\left[\dfrac{at_0^3}{3} + \dfrac{bt_0^2}{2} + ct_0\right]$

$= \dfrac{mt_0^3 a}{3} + \dfrac{mbt_0^2}{2} + mct_0$

3 Radiated power of a black body,

$P = \sigma A T^4$

where, A = surface area of the body

T = temperature of the body

and $\quad \sigma$ = Stefan's constant

When radius of the sphere is halved, new area,

$$A' = \frac{A}{4}$$

$\therefore$ Power radiated,

$P' = \sigma\left(\dfrac{A}{4}\right)(2T)^4$

$= \dfrac{16}{4} \cdot (\sigma A T^4)$

$= 4P = 4 \times 450$

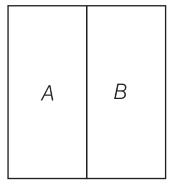
$= 1800$ W

4 $\quad K_A = 2K_B$

$\therefore \quad R_A = \dfrac{R_B}{2} \qquad \left[\because R = \dfrac{l}{KA}\right]$

Suppose, $R_A = R$,

then, $R_B = 2R$



Heat current,

$H = \dfrac{36}{R + 2R} = \dfrac{36}{3R} = \dfrac{12}{R}$

$\therefore$ Temperature difference across A

$= HR_A = \dfrac{12}{R} \times R$

$= 12°\text{C}$

5 $t = \dfrac{\rho L_f}{2KT}(d_2^2 - d_1^2)$

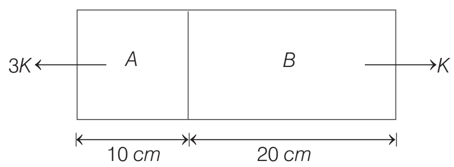
$\Rightarrow \quad t \propto (d_2^2 - d_1^2)$

$\Rightarrow \quad \dfrac{t}{t'} = \dfrac{(d_2^2 - d_1^2)}{(d_2'^2 - d_1'^2)}$

$\Rightarrow \quad \dfrac{7}{t'} = \dfrac{(1^2 - 0^2)}{(2^2 - 1^2)}$

$\Rightarrow \quad t' = 21$ h

6 $R_A = \dfrac{10}{3KA} \qquad \left[\because R = \dfrac{l}{KA}\right]$



$R_B = \dfrac{20}{KA}, \; \dfrac{R_A}{R_B} = \dfrac{1}{6}$

So, let $R_A = R$, then $R_B = 6R$

Heat current,

$H = \dfrac{\text{Temperature difference}}{\text{Total resistance}}$

$= \dfrac{35}{7R} = \dfrac{5}{R}$

Now, temperature difference across A

$= H\,R_A = \dfrac{5}{R} \times R = 5°\text{C}$

7 In steady state the amount of heat flowing from one face to the other face in time t is given by

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l},$$

where K is coefficient of thermal conductivity of material of rod

$$\Rightarrow \quad \frac{Q}{t} \propto \frac{A}{l} \propto \frac{r^2}{l} \qquad \ldots\text{(i)}$$

As the metallic rod is melted and the material is formed into a rod of half the radius

$$V_1 = V_2$$

$$\pi r_1^2 l_1 = \pi r_2^2 l_2 \qquad \left[\because r_2 = \frac{r_1}{2}\right]$$

$$\Rightarrow \quad l_1 = \frac{l_2}{4} \qquad \ldots\text{(ii)}$$

Now, from Eqs. (i) and (ii)

$$\frac{Q_1}{Q_2} = \frac{r_1^2}{l_1} \times \frac{l_2}{r_2^2}$$

$$= \frac{r_1^2}{l_1} \times \frac{4l_1}{(r_1/2)^2}$$

$$\Rightarrow \quad Q_1 = 16\,Q_2$$

$$\Rightarrow \quad Q_2 = \frac{Q_1}{16} = \frac{Q}{16} \qquad [\because Q_1 = Q]$$

8 Heat currents in both the rods are equal.

i.e. $\quad H_{CB} = H_{BA}$

or $\quad \dfrac{T_C - T_B}{\left(\dfrac{2l}{3KA}\right)} = \dfrac{T_B - T_A}{\left(\dfrac{l}{2KA}\right)}$

$\therefore \quad \dfrac{3}{2}(70 - T_B) = 2(T_B - 0)$

or $\quad T_B = 30°\text{C}$

9 $\dfrac{Q}{t} = \dfrac{KA\Delta T}{l} = \dfrac{\Delta T}{\left(\dfrac{l}{KA}\right)} = \dfrac{\Delta T}{R}$

[R = Thermal resistance]

$\Rightarrow \quad t \propto R \quad$ [$\because Q$ and ΔT are same]

$$\Rightarrow \quad \frac{t_p}{t_s} = \frac{R_p}{R_s} = \frac{R/2}{2R} = \frac{1}{4}$$

$$\Rightarrow \quad t_p = \frac{t_s}{4} = \frac{4}{4} = 1\text{ min}$$

10 From Stefan's law, the rate at which energy is radiated by sun at its surface is $P = \sigma \times 4\pi r^2 T^4$

where, r is the radius of the sun so, the power received by unit surface at a distance R from the centre of the sun.

$$I = \frac{P}{4\pi R^2} = \frac{\sigma \times 4\pi r^2 T^4}{4\pi R^2}$$

$$= \frac{\sigma r^2 T^4}{R^2}$$

$$= \frac{\sigma r^2 (t + 273)^4}{R^2}$$

11 $\dfrac{T_1 - T_2}{t} = \alpha\left[\dfrac{T_1 + T_2}{2} - T_0\right]$

Here, T_0 is the temperature of surroundings.

Substituting the values, we have

$$\frac{60 - 50}{4} = \alpha\left[\frac{60 + 50}{2} - T_0\right] \qquad \ldots\text{(i)}$$

$$\frac{40 - 30}{8} = \alpha\left[\frac{40 + 30}{2} - T_0\right] \qquad \ldots\text{(ii)}$$

Solving these two equations, we get

$$T_0 = 15°\text{C}$$

12 According to Wien's law,

$$\lambda_{\max} \propto \frac{1}{T}$$

i.e. $\lambda_{\max} T =$ constant

where, $\lambda_{\max}$ is the maximum wavelength of the radiation emitted at temperature T.

$\therefore \quad \lambda_{\max_1} T_1 = \lambda_{\max_2} T_2$

or $\quad \dfrac{T_1}{T_2} = \dfrac{\lambda_{\max_2}}{\lambda_{\max_1}} \qquad \ldots\text{(i)}$

Here, $\lambda_{\max_1} = \lambda_0$ and $\lambda_{\max_2} = \dfrac{3}{4}\lambda_0$

Substituting the above values in Eq. (i), we get

$$\frac{T_1}{T_2} = \frac{\frac{3}{4}\lambda_0}{\lambda_0} = \frac{3}{4} \text{ or } \frac{T_1}{T_2} = \frac{3}{4} \qquad \ldots\text{(ii)}$$

As we know that, from Stefan's law, the power radiated by a body at temperature T is given as

$$P = \sigma A\varepsilon T^4$$

i.e. $P \propto T^4 \quad$ ($\because$ the quantity $\sigma A\varepsilon$ is constant for a body)

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{T_1^4}{T_2^4} = \left(\frac{T_1}{T_2}\right)^4$$

From Eq. (i), we get

$$\frac{P_1}{P_2} = \left(\frac{3}{4}\right)^4 = \frac{81}{256}$$

Given, $P_1 = P$ and $P_2 = nP$

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{P}{nP} = \frac{81}{256}$$

or $\quad n = \dfrac{256}{81}$

13 According to Wien's law,

$$\lambda_m T = \text{constant}$$

where, λ_m is wavelength corresponding to maximum intensity of radiation and T is temperature of the body in kelvin.

$$\therefore \quad \frac{\lambda_{m'}}{\lambda_m} = \frac{T}{T'}$$

Given, $\quad T = 1227 + 273 = 1500\text{K},$

$T' = 1227 + 1000 + 273 = 2500\text{K},$

$\lambda_m = 5000\,\text{Å}$

Hence, $\lambda_{m'} = \dfrac{1500}{2500} \times 5000 = 3000\,\text{Å}$

14 According to Wien's law, $\lambda_m T = b$

where, $b = 2.89 \times 10^{-3}$ mK

$$\Rightarrow \quad T = \frac{b}{\lambda_m}$$

$$= \frac{2.89 \times 10^{-3}}{1.5 \times 10^{-6}}$$

$$= 2000\text{ K}$$

15 Coefficient of real expansion,

$$\gamma_r = \frac{V_2 - V_1}{V_1(T_2 - T_1)}$$

Here, $\quad V_2 = \dfrac{3}{4}, V_1 = \dfrac{2}{3}$

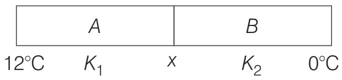
and $(t_2 - t_1) = (100 - 20) = 80°\text{C}$

$$\therefore \quad \gamma_r = \frac{\left(\frac{3}{4} - \frac{2}{3}\right)}{\frac{2}{3}(80)}$$

$$= \frac{1}{640}$$

$$= 15.6 \times 10^{-4}\,°\text{C}^{-1}.$$

16 The given situation can be shown as,



Rate of flow of heat will be equal in both the slabs

$\therefore \quad (12 - x)K_1 = K_2(x - 0)$

$$12 - x = 2x \qquad \left[\because K_1 = \frac{K_2}{2}\right]$$

$$\Rightarrow \quad x = 4°\text{C}$$

The temperature difference across slab A

$= (12 - x)$

$= (12 - 4)$

$= 8°\text{ C}$

17 Heat given for raising the temperature of W gram of water from 0°C to 100°C

$= W \times 1 \times 100$ cal

Time taken $= 10 \times 60$ s

$\therefore$ Heat given per second

$$= \frac{W \times 1 \times 100}{10 \times 60}\text{ cal}$$

Heat given out to convert W gram to steam $= W \times L$

This is the heat supplied in 55×60 s.

$\therefore$ Heat given

$$= 100 \times W \times \frac{55 \times 60}{10 \times 60} = WL$$

$$\therefore \quad L = \frac{100 \times 55 \times 60}{10 \times 60}$$

$$= 100 \times 5.5$$

$$L = 550\text{ cal/g}$$